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Peristaltic Motion of a Couple Stress Fluid of Three Dimensional Analysis with Parameter Estimation

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Abstract- The peristaltic motion of a couple stress fluid of three dimensional analysis with parameter estimation through a two dimensional flexible channel under long wave length approximation and low Reynolds number is studied. A perturbation method of solution is obtained in terms of wall slope parameter and closed form of expressions has been derived for axial velocity and transverse velocity in fluid phase. The graphical results have been presented to discuss the physical behavior of various physical parameters on axial velocity and transverse velocities. It is observed that axial velocity decreases with increase in couple stress parameter and increases as geometric parameter increases wheras transverse increases with increase in couple stress parameter and decreases as geometric parameter decreases.

Keywords: Peristaltic motion, Couple stress fluid, axial velocity, transverse velocity, Reynolds number.

1. Introduction

For the recent contribution, we refer the reader to [1—24] and the references cited therein. Peristalsis is known to be one of the main mechanisms of transport for many physiological fluids, which is achieved by the passage of progressive waves of area contraction and expansion over flexible walls of a tube containing fluid. This mechanism is found in many physiological situations like urine transport from kidney to the bladder through the ureter, swallowing food through the esophagus, movement of ovum in the female fallopian tube, vasomotor of small blood vessels, and motion of spermatozoa in cervical canal. It is also speculated that peristalsis may be involved in the translocation of water in tall trees. The translocation of water involves its motion through the porous matrix of the trees. The peristaltic transport of a toxic liquid is used in nuclear industry so as not to contaminate the outside environment. Various studies on peristaltic transport, experimental as well as theoretical, have been carried out by many researchers to explain peristaltic pumping in physiological systems. The first attempt to study the fluid mechanics of peristaltic transport is done by Latham 1966 [1]. Elsehawey EF and Mekheime Kh S (1994) [5] discussed the peristaltic flow produced by sinusoidal peristaltic wave along a flexible wall of the channel under the pressure gradient and Shapiro and Jaffrin et al.1969 [2] have studied peristaltic pumping with long wavelength at low Reynolds number.

After these studies several researchers, Raghunatha Rao T and Prasada Rao DRV (2011); Radhakrishnamacharya(2007); Elshehawey andMekheimer(1994); Elsehawey EF and El-SebaeiW (2001) have studied on peristalsis with reference to mechanical and physiological situations [9, 8, 5, 4].

The study of couple stress fluid is very useful in understanding various physical problems because it possesses the mechanism to describe rheological complex fluids such as liquid crystals and human blood. By couple stress fluid, we mean a fluid whose particles sizes are taken into account, a special case of non-Newtonian fluids. In further investigation many authors have used one of the simplification is that they have assumed blood to be a suspension of spherical rigid particles, this suspension of spherical rigid particles will give rise to couple stresses in a fluid. The theory of couple stress was first developed by Stokes (1966) [24] and represents the simplest generalization of classical theory which allows for polar effects such as presence of couple stress and body couples.

All important literature up to 2004 on peristaltic transport has been documented by M. H. Subba Reddy [23]. Ravikumar S and Siva Prasad R (2010) [20] analyzed the role of Reynolds number and wavelength in peristaltic motion of moderate amplitude, making use of perturbation method with an amplitude ratio as the perturbation parameter. Also a number of recent investigations have reported the pulsatile nature of blood flow in pulmonary arteries and different portions of mesentery. The effect of moving magnetic field on blood flow was studied by Elsehawey EF and El-Sebaei W (2001) [4], and they observed that the effect of suitable moving magnetic field accelerates the speed of blood. Srivastava, L. M. (1986) [13] considered the blood as an electrically conducting fluid constitute a suspension of red cell in plasma.

The peristaltic couple stress fluid flow through channels with flexible walls has been studied by Ravikumar et al (2010) [14]. Peristaltic flow of a couple stress fluid through porous medium in a channel at low Reynolds number studied by Raghunath Rao et al 2012 [15]. Several researchers studied

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peristaltic transport of non-Newtonian fluids Sobh, AM (2008), Raghunath Rao and PrasadRao 2011 [11, 17]. Effect of slip on peristaltic transport in an inclined wall effects has been studied by Ramanakumari AV et al (2011) [18].The present research aim is to investigate the interaction of peristalsis for the motion of a Couple stress fluid of three dimensional analysis with parameter estimation through a two dimensional flexible channel under long wave length approximation and low Reynolds number is studied. A perturbation method of solution is obtained in terms of wall slope parameter and closed form of expressions has been derived for axial velocity and transverse velocity. The computational analysis has been carried out for drawing velocity profile and parameter estimation.

Nomenclature:

d	mean half width of the channel
a	the amplitude of the peristaltic wave
c	wave velocity
λ	Wave length
t	time
p	Fluid pressure
ρ	Density of the fluid
μ	Coefficient of viscosity
η^*	Coefficient of Couple stress
ε, δ	Geometric parameters
R	Reynolds number
S	Couple stress parameter
ν	Kinematic viscosity
k_1	Permeabilit of the porous medium

2. FORMULATION OF PERISTALTIC FLOW OF A COUPLE STRESS FLUID

We consider a peristaltic flow of a couple stress fluids through two-dimensional channel bounded by flexible walls. The geometry of the flexible walls are represented by

$$y=\eta(X,t)=d+a\sin\frac{2\pi}{\lambda}(X-ct) \qquad (1)$$

a is the amplitude of the peristaltic wave, 'c' is the wave velocity, 'λ' is the wave length and 't' is the time.

Under long wavelength approximation and neglecting body forces and body couples, the equations governing the peristaltic motion of incompressible couple stress fluid for the present problem are given as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (2)$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\nabla^2 u-\eta^*\nabla^4 u-\frac{\mu}{k_1}u \qquad (3)$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\nabla^2 v-\eta^*\nabla^4 v-\frac{\mu}{k_1}v \qquad (4)$$

Where $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2},\ \nabla^4=\nabla^2\nabla^2$

u and v denote the velocities of the fluid in X,Y directions respectively

The relative boundary conditions are

$$u=0 \ \ at \ \ y=\pm\eta \qquad (5)$$

$$\frac{\partial^2 u}{\partial y^2}=0 \ \ at \ \ y=\pm\eta \qquad (6)$$

$$v=0 \ \ at \ \ y=0 \qquad (7)$$

Equation (5) represents no slip on the boundary, (6) indicates the boundary condition related to couple stress fluid and (7) shows velocity at the center of the channel.

Introducing a wave frame *(x, y)* moving with velocity *c* away from the fixed frame *(X, Y)* by the transformation

$x=X-ct,\ \ y=Y,\ \ u=U-c,\ \ v=V,\ \ p=P(X,t)$

Using the following the non-dimensional variables

$$x'=\frac{x}{\lambda},\ y'=\frac{y}{d},\ u'=\frac{u}{c},\ v'=\frac{v}{c\delta},\ t'=\frac{ct}{\lambda}\ \ \eta'=\frac{\eta}{d},\ p'=\frac{pd^2}{\mu c\lambda} \qquad (8)$$

These equations motion and boundary conditions reduces to dimensionless form

$$y=\eta(x)=1+\varepsilon\sin 2\pi x \qquad (9)$$

$$R\delta\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\left(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) -S\left(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+D^{-1}u \qquad (10)$$

$$R\delta^3\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\delta^2\left(\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) -s\delta^2\left(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+D^{-1}\delta^2 v \qquad (11)$$

Where

$$\varepsilon=\frac{a}{d}\ \ and\ \ \delta=\frac{d}{\lambda}\ \ R=\frac{cd}{\nu}\ \ s=\frac{\eta^*}{\mu d^2}\ \ D=\frac{k_1}{d^2}$$

The corresponding dimensionless boundary conditions are

$$u=0 \ \ \ at \ \ y=\pm\eta \qquad (12)$$

$$\frac{\partial^2 u}{\partial y^2}=0 \ \ at \ \ y=\pm\eta \qquad (13)$$

$$v=0 \ \ \ at \ \ y=0 \qquad (14)$$

3. PERTURBATION SOLUTION

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We seek perturbation solution in terms of small parameter δ as follows

$$u = u_0 + \delta u_1 + \delta^2 u_2 + ... \qquad (15)$$

$$v = v_0 + \delta v_1 + \delta^2 v_2 + ... \qquad (16)$$

Substituting equations (12) to (14) in equations (9) to (11) and collecting the coefficients of various powers of δ

The zeroth order equations are

$$s\frac{\partial^4 u_0}{\partial y^4} - \frac{\partial^2 u_0}{\partial y^2} + D^{-1}u_0 = -\frac{\partial p_0}{\partial x} \qquad (17)$$

$$\frac{\partial p_0}{\partial y} = 0 \qquad (18)$$

The corresponding dimensionless boundary conditions are

$$u_0 = 0 \quad at \;\; y = \pm\eta \qquad (19)$$

$$\frac{\partial^2 u_0}{\partial y^2} = 0 \;\; at \;\; y = \pm\eta \qquad (20)$$

$$v_0 = 0 \quad at \;\; y = 0 \qquad (21)$$

On solving the equations (17) and (18) subject to the conditions (19) to (20), we get

$$u_0 = A2 + A3\cosh(\beta y) + A4 y^2 \qquad (22)$$

$$v_0 = A5 y + A6\sinh(\beta y) \qquad (23)$$

where

$$A_1 = -\frac{\partial p_o}{\partial x} \quad A2 = C2\left[1-\eta^2\right] \quad A3 = -A1 s\,\mathrm{sech}[\beta\eta] \quad A4 = \frac{A1}{2}$$

$$A_5 = 4\pi\varepsilon C_2\cos 2\pi x,\; A_6 = 2\pi\varepsilon A_1 s\cos 2\pi x\frac{\sinh[\beta\eta]}{\cosh^2[\beta\eta]},\; \beta = \frac{1}{\sqrt{s}}$$

$$C_1 = 0,\, C_2 = A_1\left(1-\eta^2\right), \quad C_3 = C_4 = -\frac{A_1 s}{2\cosh\left[\frac{\eta}{\sqrt{s}}\right]}$$

4. Results and Discussion

In this section, we have presented the graphical results of the solutions axial velocity u_0, transverse velocity v_0 .The axial velocity u_0 and its 3- dimension effects is shown in Figures (1) to (6), In fig.(1) the axial velocity u_0 is exhibited for different values of A_1 in the region y = 0 to y = 1. It is found that the velocity u_0 is decreases as A_1 increases and in figure (2) the three dimensional view of the axial velocity u_0 is exhibited. In fig (3) the axial velocity u_0 is exhibited for different values of s in the region y = 0 to y =1. It is found that the velocity u_0 decreases as couple stress parameter s increases. In fig (5) to (6), the axial velocity u_0 is exhibited for different values of ε in the region y = 0 to y = 1. It is observed that the axial velocity u_0 increases for different values of geometric parameters (see Table.1). The transverse velocity v and its effect is shown in Figures (7) to (9), In fig.(7) the transverse velocity v_0 is exhibited for different values of A_1 in the region y = 0 to y = 1. It is found that the velocity v_0 increases as A_1 increases. In fig (8) the transverse velocity v_0 is exhibited for different values of s in the region y = 0 to y =1. It is found that the velocity v_0 increases as couple stress parameter s increases. In fig (9) the transverse velocity v_0 is exhibited for different values of ε in the region y = 0 to y = 1. It is observed that the transverse velocity v_0 decreases as ε increases (see Table.1).

Table.1: Effects of parameters estimation on u and v

S.No	Parameters	Value used	Remarks
1	A_1	A_1 =1, 2, 3, 4, 5	u_0 decreases as A_1 increases
2	s	s=0.2,0.22,0.24,0.26,0.28	u_0 decreases as s increases
3	ε	ε =0.1, 0.2, 0.3, 0.4, 0.5	u_0 increases as ε increases
4	A_1	A_1 =1, 2, 3, 4, 5	v_0 inecreases as A_1 increases
5	s	s=0.2,0.22,0.24,0.26,0.28	v_0 increases as s increases
6	ε	ε =0.1, 0.2, 0.3, 0.4, 0.5	v_0 decreases as ε increases

5. FIGURES

Fig.1: Effect of A1 on u0 when s = 0.20, ep = 0.1, beta = 0.5

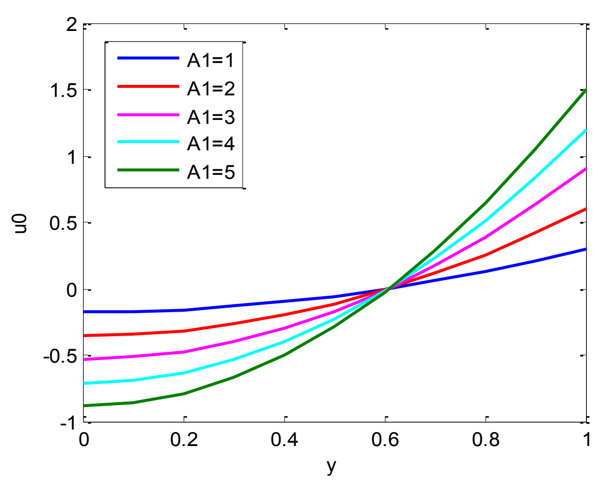

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Fig.2: Three dimension effects of A1 on u0 when s = 0.20, ep = 0.1, beta = 0.5

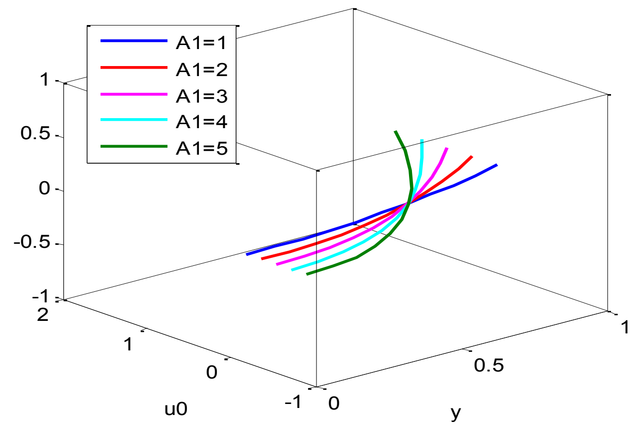

Fig.3: Effect of s on u_0 when A1 = 1, ep = 0.1, beta = 0.5

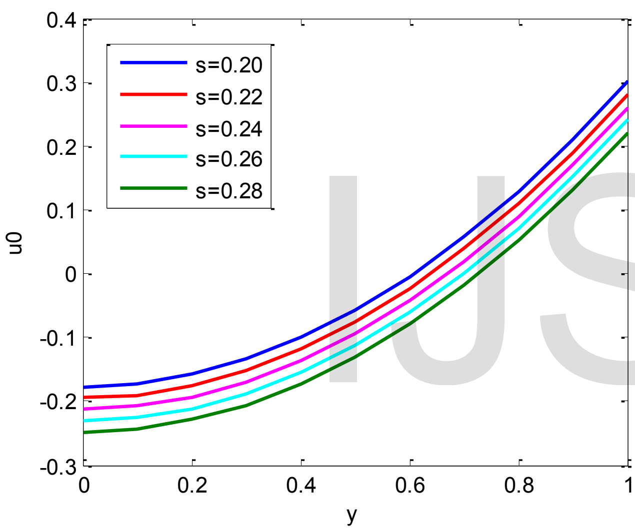

Fig.4: Three dimension effects of s on u_0 when A1=1, ep = 0.1, beta = 0.5

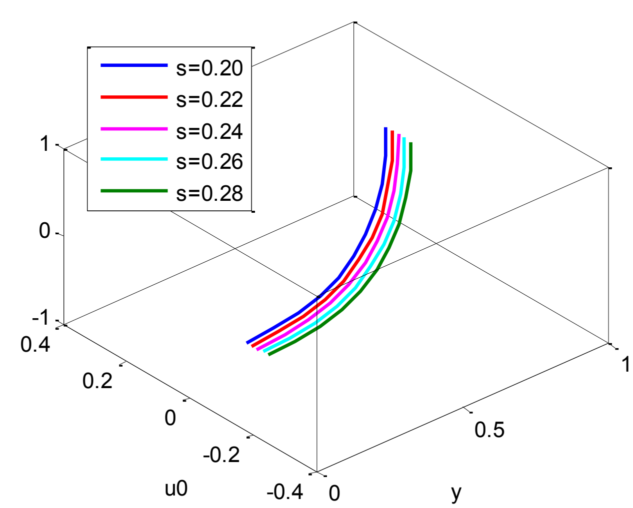

Fig.5: Effect of ep on u_0 when s = 0.20, A= 1, beta = 0.5

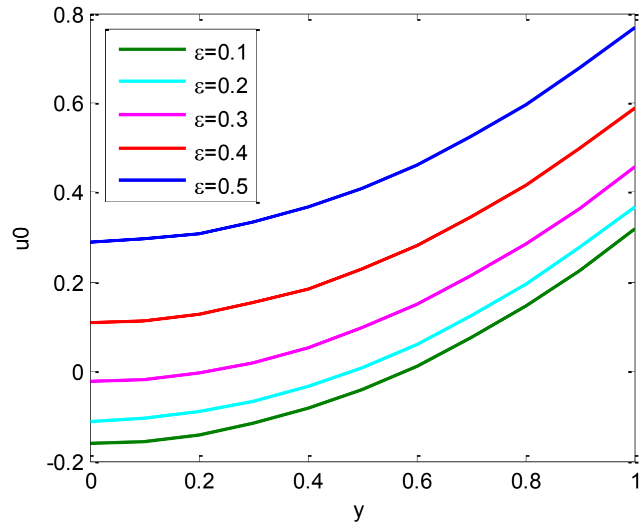

Fig.6: Three dimension effects of ep on u_0 when s = 0.20, A1= 1, beta = 0.5

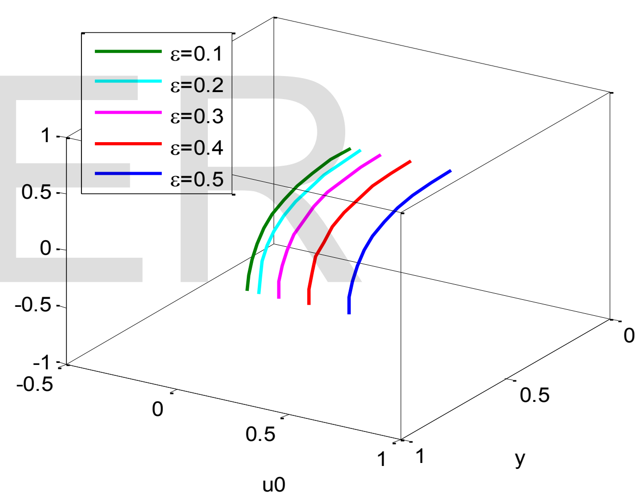

Fig.7: Effect of A1 on v_0 when s = 0.20, ep = 0.1, beta = 0.5

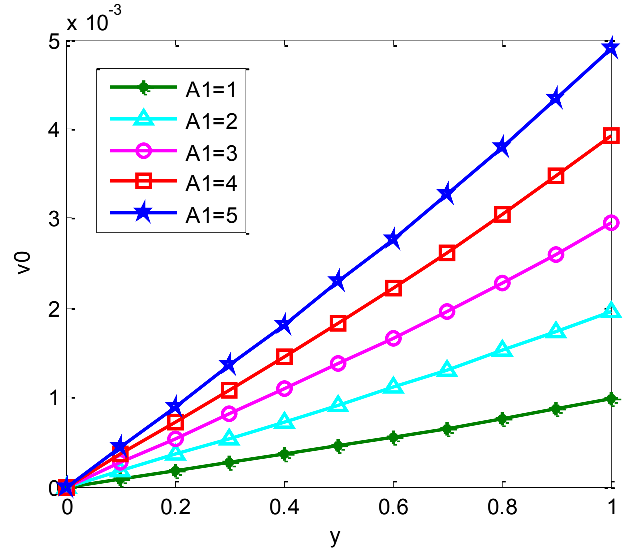

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Fig.8: Effect of s on v_0 when A1=1, ep = 0.1, beta = 0.5

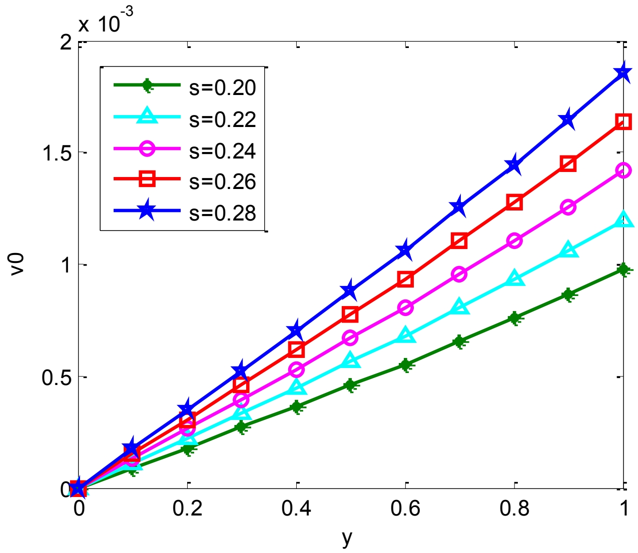
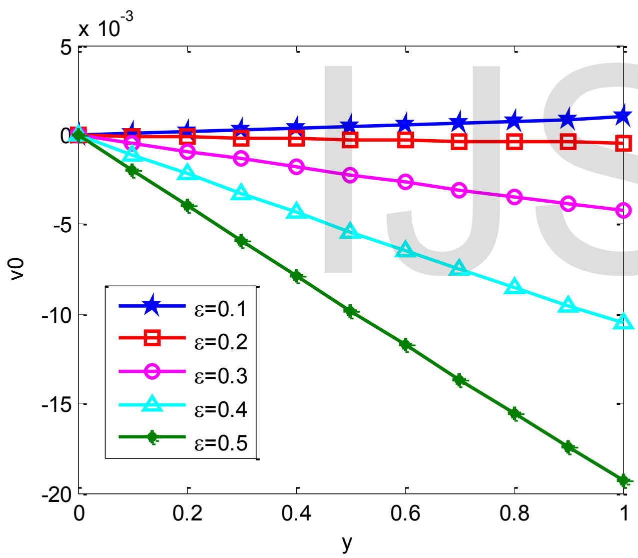


Fig.9: Effect of ep on v_0 when s = 0.20, A1= 1, beta = 0.5



6. CONCLUSION

In this paper we studied the interaction of peristalsis for the motion of a Couple stress fluid of three dimensional analysis with parameter estimation through a two dimensional flexible channel under long wave length approximation and low Reynolds number is studied. A perturbation method of solution is obtained in terms of wall slope parameter and closed form of expressions has been derived for axial velocity and transverse velocity. The computational analysis has been carried out for drawing velocity profile and parameter estimation. We conclude that that the axial velocity decreases with increase in couple stress parameter and A_1 whereas axial velocity increases as geometric parameter ε increases. But in the case of transverse velocity it increases as couple stress parameter and A_1 increases and decreases when geometric parameter ε increases.

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